ICONHIC 2016
1st International Conference on Natural Hazards & Infrastructure
28-30 June, 2016, Chania, Greece

Inverse analysis of laterally loaded monopiles subjected to large cyclic loading: development of a Bouc-Wen type soil reaction model

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ABSTRACT

Offshore structures are located in a harsh environment subjected to million loading cycles due to wave and wind loading, which is cyclic in nature. Particularly in the case of pile foundations (monopiles, anchor-piles), it has been shown that even a low amplitude loading of many cycles can decrease the foundation stiffness and lead to accumulation of permanent rotations, which may compromise the operation of the structure. The analysis of the 3D problem, through numerical tools, like finite elements is not feasible at least with the current computing technology. In order to alleviate these issues, an alternative numerical analysis technique is developed, simulating a pile employing a nonlinear Winkler-type model. The soil is represented by lateral translational nonlinear springs of the Bouc-Wen type, while a modification factor is introduced in the differential Bouc-Wen governing formula in order to reproduce the degradation of stiffness and the accumulation of rotation due to the large number of loading cycles. The Bouc-Wen type Winkler model is controlled by 6 parameters which are calibrated against centrifuge tests and FE analysis results via optimizations algorithms, capturing the long-term cyclic response satisfactorily. Additionally, the computational time is reduced significantly, allowing analyzing the problem of a pile subjected to 50 loading cycles in less than a minute, while the relevant 3D problem solved via finite elements demands more than 12 hours.

Keywords: pile, Bouc-Wen, cyclic loading, lateral loading, optimization algorithm

INTRODUCTION

The long term performance of the monopiles is a significant issue that is not fully investigated up to now. The OWT are subjected to millions of loading cycles due to environmental actions and machine vibration induced loading. Except the large loads that are applied during storms, smaller long term loads can violate the serviceability limit state (SLS). These loads cause rocking of the monopile resulting in severe disturbance of the surrounding soil. This phenomenon can significantly affect the lateral stiffness of the OWT-foundation system, but can also lead to accumulation of permanent rotations. The change in stiffness could shift the Eigen mode frequencies of the wind turbine towards the vibration frequencies resulting in resonance. On the other hand, the accumulation of permanent rotation can harm the functionality of the OWT as the mechanical equipment is sensitive to displacements.

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Until now, the above described issue is not addressed by the existing design standards, i.e., DNV (2010) and API (2007). The recommended design *p-y* curves are based on expressions best suited for conventional pile foundations and trivial loading conditions rather than large-diameter offshore piles subjected to large cyclic loading. To bridge this gap, a simple constitutive model for lateral soil reaction is proposed which is a reformulation of that originally proposed by Gerolymos et al. (2005; 2009), capable of describing the response for a very large number of loading cycles. The developed modified BWGG model is a versatile one-dimensional action–reaction relationship, capable of reproducing an almost endless variety of stress–strain or force–displacement or moment–rotation relations, monotonic as well as cyclic The goal is to reduce the computational cost and to develop an engineering tool for estimating the long-term performance of laterally loaded piles.

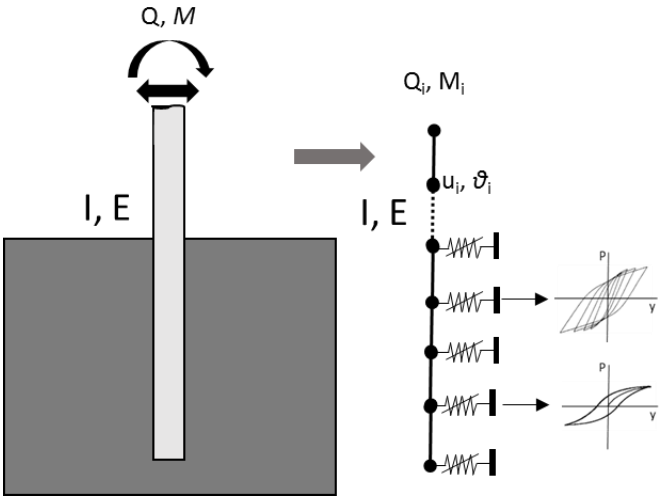

Figure 1. Schematic illustration of the analyzed problem.

METHODOLOGY

Numerical model and constitutive equations

In order to simplify the problem and to reduce the computational cost of the analysis we analyze the 2D problem, while the vertical loads and displacements are ignored (Fig. 1). The pile and the tower are considered as linear-elastic and are modelled through Bernoulli elements with stiffness matrix:

$$\mathbf{K} = EI \begin{bmatrix} \frac{12}{l^3} & \frac{6}{l^2} & -\frac{12}{l^3} & \frac{6}{l^2} \\ \frac{6}{l^2} & \frac{4}{l} & -\frac{6}{l^2} & \frac{2}{l} \\ \frac{12}{l^3} & -\frac{6}{l^2} & \frac{12}{l^3} & -\frac{6}{l^2} \\ \frac{6}{l^2} & \frac{2}{l} & -\frac{6}{l^2} & \frac{4}{l} \end{bmatrix} \tag{1}$$

Where: **K** is the stiffness matrix of an element with length *l*, *E* the Young's modulus of the tower and *I*, the cross-section moment of inertia. The soil is replaced by lateral Winkler springs, the behaviour of which to lateral loading is described by the 'BWGG' model (Gerolymos and Gazetas 2005; 2006, Gerolymos et al. 2009). According to this, the lateral soil resistance p_y per unit depth of the pile is expressed as the resultant of two components:

$$p_x = \alpha p_y + (a-1)p_y\zeta \tag{2}$$

In which: p_x is the soil reaction; p_y is the ultimate soil reaction; a is a parameter that controls the post-yield stiffness; and ζ is a hysteretic dimensionless quantity controlling the nonlinear behavior of the lateral soil resistance. The parameter ζ is governed by the following differential equation, with respect to time, t.

$$\frac{d\zeta}{dt} = \frac{1}{u_y}\left[1 - |\zeta|^n[b + gsign(du\zeta)]\right]\frac{du}{dt} \tag{3}$$

In Equation 3, the parameters b, g and n are dimensionless quantities that control the shape of the loading loops, and u_y is the value of the lateral displacement at initiation of yielding in the soil at the specific depth. For the calculation of the parameter u_y the variation of the shear modulus (G) with the depth (z) is taken into account. Hence, the parameter u_y is calculated according to Equation 4, in which, z is the depth; p_y is the ultimate soil reaction and m is a dimensionless parameter which controls the variation of shear modulus (G) with the depth (z).

$$u_y = \left(\frac{p_y}{K}\right) z^{(1-m)} \tag{4}$$

Differentiating Equation (2) with respect to u and using Equation (3) yield:

$$\frac{dp_x}{du} = K_x = aK + (a-1)K\left[1 - |\zeta|^n[b + gsign(du\zeta)]\right] \tag{5}$$

which describes the tangent stiffness of lateral soil reaction K_x. K is the initial (elastic) stiffness at zero displacement.

Constitutive Equations accounting for cyclic loading

As mentioned before, the cyclic loading of piles restructures the soil. The foundation stiffness gradually changes with the number of cycles, resulting in accumulation of permanent rotations. To reproduce this behavior, the Bouc-Wen (1971, 1976) governing equation is modified by replacing ζ with ζ^* in Equations 3 and 5. Parameters α, β (Equation 6) are dimensionless parameters influenced by the characteristics of the cyclic loading and its amplitude, while N is the number of loading cycles.

$$\zeta^* = \frac{\zeta}{1 + \alpha N^\beta} \tag{6}$$

$$\frac{d\zeta}{dt} = \frac{1}{u_y}\left[1 - |\zeta^*|^n[b + gsign(du\zeta)]\right]\frac{du}{dt} \tag{7}$$

$$\frac{dp_x}{du} = K_x = aK + (a-1)K\left[1 - |\zeta^*|^n[b + gsign(du\zeta)]\right] \tag{8}$$

Evidently, Equation 3 is of hysteretic rather than viscous type, hence, its solution is not frequency dependent. Two different numerical integration schemes were applied for solving Equations (7) and (8): the central finite difference and the fourth-order Runge-Kutta. The negligible superiority of Runge-Kutta in improving the

accuracy compared to the time-effectiveness of the finite difference method was the main reason that the latter was finally adopted as the integration scheme. Figure 2 compares the response in terms of displacement (in m) at the top of the pile as a function of the finite element length (in m) and time discretization step.

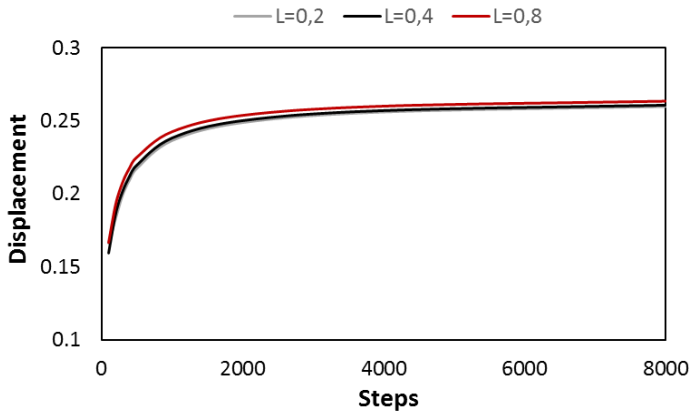

Figure 2. Sensitivity analysis in terms of horizontal displacement (in m) at the top of a pile subjected to horizontal loading at its head. The parameters are: (a) the finite element length (in m), and (b) the number of time steps

Calibration Methodology

The calibration of the model parameters was based on matching the calculated with the measured force-displacement curve at the pile head with the help of suitable heuristic optimization algorithms available in MATLAB. Regarding the calibration possess, 6 parameters had to be determined (K, m, p_y, n, α and β); Parameters α and β govern the cyclic behavior in terms of densification, plastic shake down or relaxation. The optimization procedure includes 3 steps. Initially, for the estimation of K and m the computed response is matched with the corresponding measured one in terms of force-displacement at the top of the pile (minimization function) at very small deformations (quasi-elastic regime); in this step, high values are assigned to p_y and n to secure linear behavior. Then, having calibrated K and m, the estimation of p_y and n is achieved by fitting the calculated to the measured response at very large (close to failure) displacements. Finally, the goal function to be minimized for computing the cyclic loading parameters α and β, is the displacement of the pile head at the pivot (reloading-unloading or unloading-reloading) points.

APPLICATION

Herein, the model is calibrated against a series of centrifuge tests of piles in sand and its corresponding 3D finite element analysis results (Giannakos et al., 2012). The centrifuge tests were conducted for the dissertation of Rosquoët (2004) at the Laboratoire Central des Ponts et Chaussées (LCPC). The tests were performed on a single pile of length L = 15.2 m, diameter D = 0.72 m and thickness t = 0.12 m in prototype scale, subjected to cyclic horizontal loading. The centrifuge models, 1/40 in scale, involved pile head loading for three different force time histories: i) 12 cycles from 960 kN to 480 kN, ii) 12 cycles from 960 kN to 0 kN, iii) 12 cycles from -960 kN to 960 kN, which presented in Figure 3.

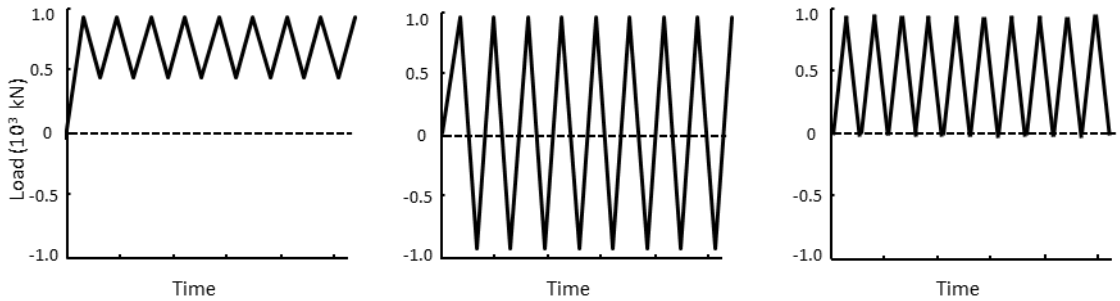

Figure 3. Lateral load histories of the 3 tests

Regarding the soil, the unit weight and the relative density of the sand were measured to be $\gamma_d \approx 16.5 \pm 0.04$ kN/m^3 and $D_R = 86\%$ respectively. Laboratory results from (drained and undrained) torsional and direct shear tests on sand reconstituted specimens indicated mean values of peak and critical state angles of φ_p = 41.8^o and φ_{cv} = 33° , respectively.

Table 1 presents the values of the model parameters after calibration with results from FE analyses (Giannakos et al, 2012). Comparison with the FE model and the centrifuge tests is illustrated in Figures 4 and 5 in terms of force-displacement loops at the head of the pile. Evidently, the calibrated Winkler model is capable of reproducing the measured and the calculated by the FE model response with satisfactory accuracy. Each analysis was carried out without re-adjustment of K, m, p_y and n. However, re-adjustment of parameters α and β, which control the cyclic response, is necessary for capturing the response under different load profiles. The value of parameter m = 0.488 reflects the versatility of the applied optimization procedure to yield results consistent with a priori known properties of the problem. That is for example the increase of shear modulus (G_0) with the square root of depth ($\sqrt{z}$). The predictions with the optimized Winkler model accumulated displacements are in well agreement with those from the numerical analyses and the centrifuge experiments. In general, the model is shown to be capable of predicting some complicated features of the measured and the computed response regarding the asymmetric load profiles (Figure 4), arising from counteracting phenomena such as strength relaxation and stiffness hardening of the pile with cyclic loading, as well as the reduction in hysteretic damping possibly due to the development of a relaxation zone around the upper part of the pile. The decrease of displacement at load reversals, but at a decreasing rate, until plastic shake down is reached, is also efficiently captured by the developed Winkler model (Figure 5).

Table 1. Calibrated model parameters against results from finite element analysis

		960 to 480 kN	**960 to 0 kN**	**-960 to 960 kN**
K	kN/m^3		112768	
m	-		0.488	
p_y	kN/m^2		233	
n	-		0.5	
α	-	0.269	0.033	0.263
β	-	0.422	0.414	0.081

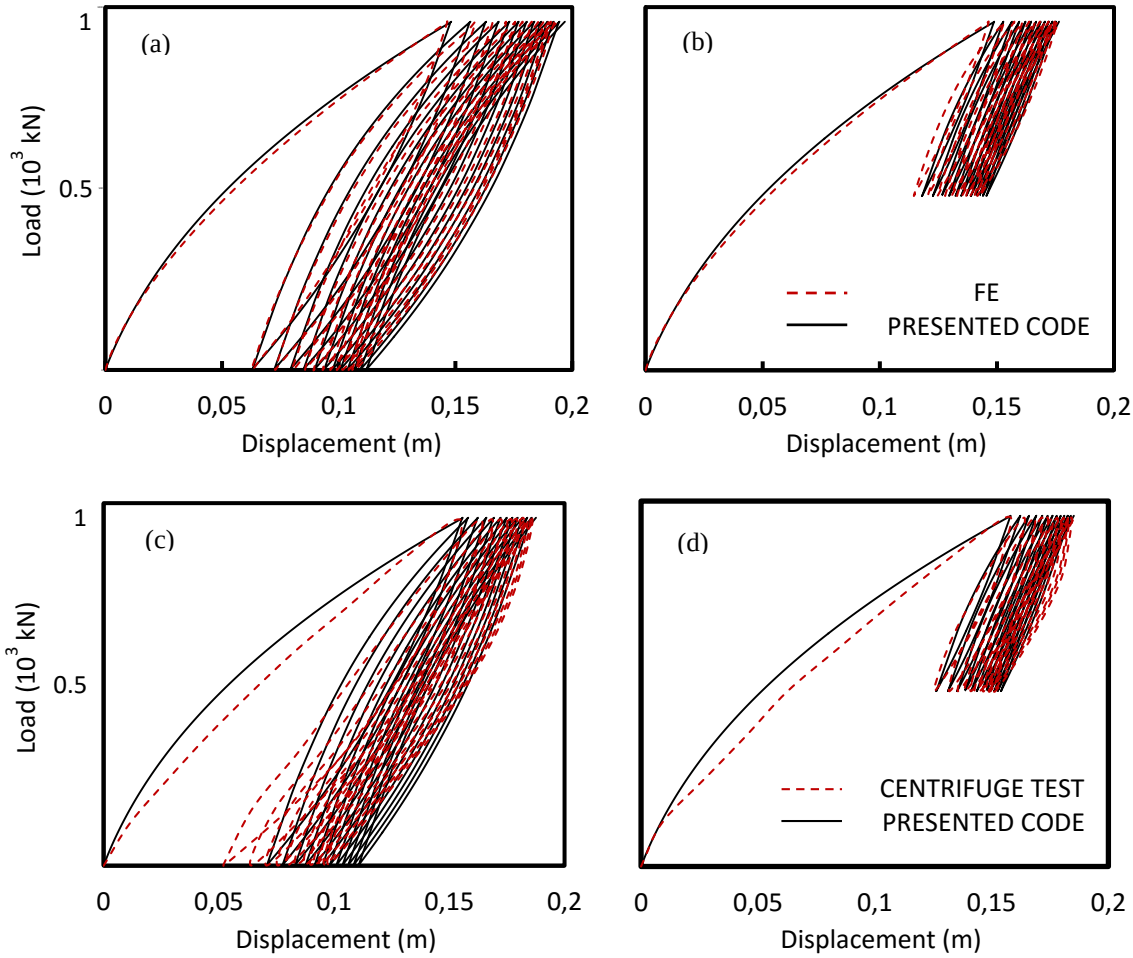

Figure 4. Comparison of horizontal force-displacement loops at the pile head, computed with the developed Winkler model calculated from finite element analysis (a), (b), and measured in centrifuge tests (c), (d). Results are given for two asymmetric loading profiles

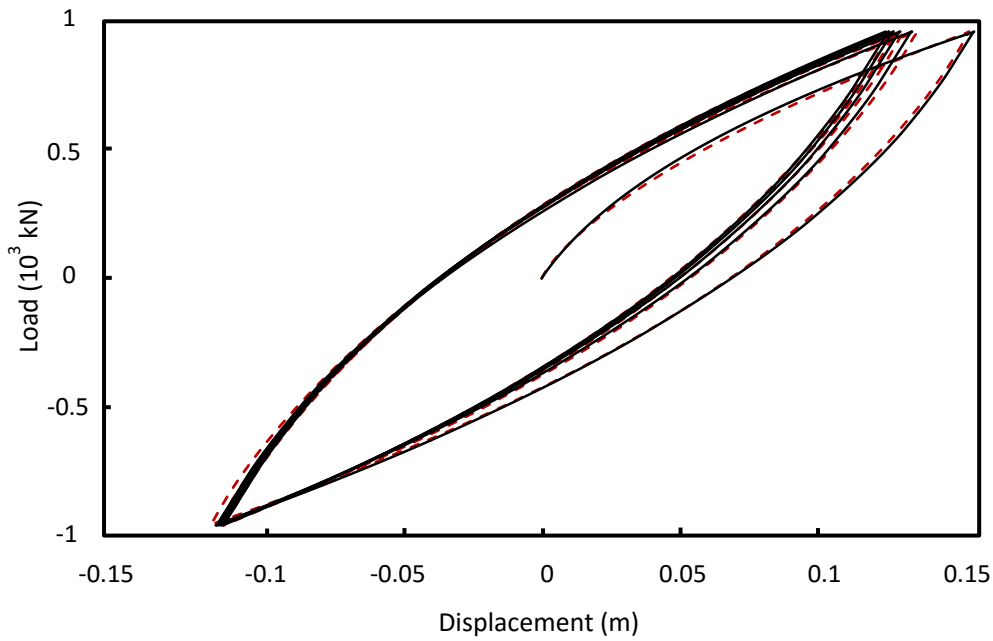

Figure 5. Comparison of horizontal force-displacement loops at the pile head, computed with the developed Winkler model and calculated from finite element analysis for symmetric loading

CONCLUSIONS

A Winkler type model for the lateral response of laterally loaded piles to large cyclic loading is presented. The modeling of soil reaction is controlled by 6 parameters; k, m, p_y, n, α and β. The first four parameters are directly related to mechanical properties of soil, while α and β control the behavior under cyclic loading, and depends on the detailed characteristics of the loading history (e.g. LeBlanc et al., 2010a). For the calibration of the model parameters a simple three-step optimization procedure was developed, aiming at matching the computed response with measured results from centrifuge tests and FE analysis. It was shown that the model is capable of simulating the measured response with remarkable accuracy for a variety of loading conditions. Moreover, the developed numerical code reduces significantly the computational cost, making feasible to analyse up to 100 loading cycles in a few minutes, contrary to FE modelling in which 10 loading cycles demand more than 12 hours of computational time.

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